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CERTAIN FORMULATIONS OF THREE-DIMENSIONAL OPTIMIZATION PROBLEMS IN HYPERSONIC AERODYNAMICS

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The fundamental configuration of a prospective hypersonic aircraft in which the active balance of forces is created by a direct-flow air-breathing jet engine in liquid hydrogen with supersonic combustion is dictated by its specific functioning conditions. Thus, in order to ensure the intake of air from the atmosphere during flight in a rarefied medium the air-intake system should have a reasonably wide capture area, which will in fact differ very little from the middle cross section of the whole aircraft. The nozzle (second element in the engine system) should also have large dimensions. These engine elements should make a specific contribution to the aerodynamics of the aircraft as a whole; they are characterized by large areas immersed in the flow, on which the function of carrying surfaces will to a certain extent be imposed. Hence we have the necessity of asymmetry in the configurations of such surfaces and the associated essentially three-dimensional character of the perturbed flow.

Let us consider the following presentation of the fundamental problem: in a three-dimensional space we have two specified arbitrary closed contours l_1 and l_2 (Fig. 1); the isobars of the unknown flow are based on these contours, the pressures on the latter being specified as p_1 and p_2, respectively. It is required to find the stream surface passing through both contours and optimizing a certain integrated force characteristic of the unknown surface. The problem is made specific by giving the functional of the mechanical (force) action.

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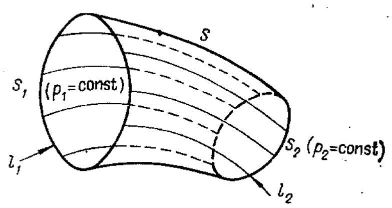

Fig. 1

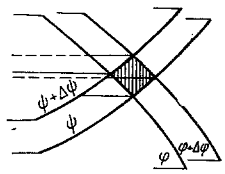

Fig. 2

Practical hypersonic calculations make extensive use of approximate and semiempirical methods, the chief of which include Newton's theory, the method of rarefaction waves, and the tangential cone method [1] (we shall subsequently call these the "principal approximate methods"). These methods are directed at solving the problem of the pressure distribution over the surface of the body in the flow. The solution of this problem allows us to approach the question of optimizing the shape of the body as regards force characteristics. The optimization problem demands reasonable simplicity of the algorithm for calculating the pressure at various points of the surface so as to ensure that the formulation of the problem should be nonformal. The principal approximate methods possess this simplicity, but encounter considerable difficulties when extended to spatial problems and when establishing methods of making a reasonable estimate of their accuracy. Since these methods start from essentially differing physical prerequisites, the possibility of giving them a unique analytical representation on the basis of some unifying index (which would be especially useful when considering possible versions of combined methods) is not entirely obvious. Thus we may distinguish three main questions in the theory of the principal approximate methods: 1) the development of recommendations as to the extension of the methods to spatial problems; 2) a study of the possibility of making a reasonable estimation of accuracy and constructing higher approximations; 3) the search for a universal analytical form describing the methods in question.

This paper will be devoted to a consideration of these problems in relation to the question of optimization just discussed.

§1. To avoid having to study each method separately, let us start by obtaining a satisfactory answer to the third question. At the same time we shall propose a certain formal grounding for the three methods.

Let us consider the steady-state, axisymmetrical flow of a nonviscous and non-heat-conducting gas with arbitrary thermodynamic properties and assume the following notation: x, y, geometrical coordinates in the plane of the axial cross section; p, pressure; ρ, density; h, heat content; ψ, stream function; w, velocity modulus; u, v, projections of the velocity on the x and y axes; M, Mach number; ϑ, inclination of the velocity vector to the symmetry axis; S, entropy. All the dimensional quantities are referred to the corresponding maximum values. We shall regard the heat content h as a specified function of pressure and entropy (equation of state): $h=h(p, S)$.

Let us introduce the pressure p and stream function ψ as independent variables [2]. We shall then have

$$d\psi = \rho y(udy - vdx) = \rho y[u(y_p dp + y_\psi d\psi) - v(x_p dp + x_\psi d\psi)]$$

or

$$(\rho y u y_\psi - \rho y v x_\psi - 1)d\psi = \rho y(v x_p - u y_p)dp,\ 1/\rho = h_p.$$

In view of the independence of dp and dψ,

$$u y_\psi - v x_\psi = h_p/y;\ v x_p - u y_p = 0. \qquad (1.1)$$

The Euler equation in the variables (p, ψ) is written in the form

$$y y_\psi + u_p = 0.$$

This equation will be satisfied identically if we assume that

$$y^2/2 = \sigma_p;\ u = -\sigma_\psi, \qquad (1.2)$$

where $\sigma(p, \psi)$ is an arbitrary function. Solving (1.1) for the derivatives x_p, x_ψ and eliminating x by cross differentiation, we obtain an equation containing only one function σ:

$$2(1-h)(\sigma_{p\psi}^2 - \sigma_{pp}\sigma_{\psi\psi}) - \left[h_S S'(\psi)\sigma_\psi + h_p \frac{2(1-h-\sigma_\psi^2)}{2\sigma_p}\right]\sigma_{pp} + 2h_p\sigma_\psi\sigma_{p\psi} + h_p^2 + [2(1-h)-\sigma_\psi^2]h_{pp} = 0. \qquad (1.3)$$

If the flow is isentropic, we have $S'(\psi) \equiv 0$, and the first term in the coefficient of σ_{pp} in Eq. (1.3) vanishes. The second term in this coefficient is due to axial symmetry. Thus in the case of plane isentropic flows the coefficient of σ_{pp} is equal to zero.

We shall seek a solution to Eq. (1.3) in the form of a linear function of the stream function

$$\sigma = a(p) + b(p)\psi. \tag{1.4}$$

In the case of isentropic flow, exact solutions of this form do in fact exist. By the direct substitution of (1.4) into (1.3) we obtain the following ordinary second-order differential equations for the functions a (p) and b(p):

$$b'' = b'L(b); \; a'' = a'L(b). \tag{1.5}$$

Here the primes denote derivatives with respect to p and L(b) signifies the differential operator:

$$L(b) = \frac{2h_p}{1-h-b^2}\left[2(1-h)(b')^2 + 2h_p bb' + \frac{1}{2}h_p^2 + (1-h-b^2)h_{pp}\right].$$

If a and b satisfy Eqs. (1.5), then a function σ of form (1.4) will describe conic flows, since it follows from (1.2) that p and ϑ are conserved on the same lines, which by virtue of (1.1) and the definition of ψ are straight. Let us consider the case of plane flows in more detail. In Eqs. (1.5) the terms with the leading derivatives are then absent and the system (1.5) is satisfied by an arbitrary function a (p) if b(p) is a solution of L(b) = 0.

The latter equation is easily integrated. The result takes the form

$$b = w \sin[\nu(w) + c], \quad \nu(w) = \int \frac{\sqrt{M^2-1}}{w}\,dw,$$

where c is an arbitrary constant; ν (w) is the Prandtl – Meyer function. Thus in the plane case Eq. (1.3) has an exact particular solution

$$\sigma = \pm\sqrt{1-h}\sin(\nu + c)\psi + a(p),$$

having an arbitrary function a (p) and an arbitrary constant c. This solution describes a simple wave, since the velocity components retain constant values on the isobars.

Thus if the function σ depends linearly on the stream function in the axisymmetrical case, this will signify conical flow and in the plane case, a simple wave.

Instead of h(p, S) let us introduce the function H(p, S) from

$$1 - h = \Phi^2(S)(1 - H), \tag{1.6}$$

where Φ (S) is an as-yet arbitrary function. Since $S = S(\psi)$, $\eta = \int \Phi(S)d\psi$ will be a function of ψ only. After eliminating the function h from Eq. (1.3) by means of (1.6) the form of this equation will remain the same. Simple conversion of the derivatives shows that h is simply replaced by H and ψ by η ("substitution principle" [3]). We shall therefore subsequently make no distinction between the variables ψ and η.

Let us consider the degenerate case in which h depends solely on S and not on the pressure. The equation $h_p = 1/\rho = 0$ may be treated as meaning infinite density in the stream. The functional determinant

$$D\left(\frac{x, y}{p, \psi}\right) = -\frac{h_p\sigma_{pp}}{H\sigma_p\sqrt{1-h-\sigma_\psi^2}} = 0;$$

i.e., the flow region degenerates into a line. In Eq. (1.6) we may put H = 0; then $\Phi(S) = \sqrt{1-h}$. For the deformed variable η we retain the notation ψ. Equation (1.3) reduces to the equation of developing surfaces

$$\sigma_{pp}\sigma_{\psi\psi} - \sigma_{p\psi}^2 = 0, \tag{1.7}$$

in which

$$\sigma_\psi = -\cos\vartheta. \tag{1.8}$$

It follows from (1.7) that

$$\sigma_\psi = f(\sigma_p),$$

where f is an arbitrary function. Subsequent integration of the equation, for example, by the complete-integral method, leads to an analytical formulation of the Newton theory with the Buseman correction for centrifugal forces. The usual Newton approximation is obtained from (1.8), in which we must consider that the ϑ on the right-hand side depends solely on the pressure. Thus $\sigma = -\cos \vartheta \psi + b(p)$, where $b(p)$ is an arbitrary function depending on the shape of the body. Hence the Newton approximation is also described by a relationship of form (1.4).

In general, according to (1.2), $\sigma = -\int u d\psi$, where the indefinite integral is taken along an isobar. It follows that σ represents the flow of momentum through the line p=const in the direction of the symmetry axis. For the principal approximate methods this quantity depends linearly on the mass of gas having momentum σ.

This characteristic will subsequently be used for generalizing the methods to spatial problems and constructing higher approximations by allowing for quadratic and higher powers of the mass of gas in the equations for the flow of momentum through the isobars.

Depending on the method of determining the coefficients, different approximate methods are obtained. For example, if we require that a linear binomial should satisfy the equation of plane flows, then the remaining degree of freedom only allows us to satisfy the boundary condition of impermeability of the surface immersed in the flow. The construction of $\sigma(p, \psi)$ in such an approximation gives a result coinciding with the method of rarefaction waves. If we require that the linear function (1.4) should satisfy Eq. (1.3) exactly, then the a (p) and b(p) so found will give an analytical representation of the method of tangential cones. The integration constants of Eq. (1.5) will depend on the angle ϑ at the surface. Finally, if we require that all the boundary conditions should be satisfied exactly in the problem of the circumfluence of a medium with an equation of state $h=h(S)$, then this will determine the functions a (p) and b(p), subject to the additional assumption that the pressure depends solely on the local angle of inclination of the surface.

Let us consider the possibility of specifying a quadratic dependence of σ on ψ along the lines p=const: $\sigma = a + b\psi + c\psi^2$, where a, b, c are certain functions of pressure. After placing this type of function σ in Eq. (1.3) the left-hand side will be a polynomial of the fourth degree in ψ ($S'(\psi) \equiv 0$). If we equate the coefficients of this polynomial to zero, we obtain an overdetermined system of five equations for the three functions of pressure a, b, and c. The system is compatible, but its solution only has freedom with respect to two constants:

$$a = \text{const}; \; b = -\sqrt{1-h}; \; c = c\sqrt{1-h},^*$$

where c is an arbitrary constant. Since σ is defined to the accuracy of an arbitrary constant, the constant a is inessential. Thus the solution of Eq. (1.3) constituting a polynomial of the second degree in ψ may be written in the form

$$\sigma = -\sqrt{1-h}\,\psi(1 - c\psi).$$

This relationship describes flow from a spherical source.

§2. In order to study possible transitions to spatial problems let us take as independent variables the pressure p and two stream functions φ and ψ; we shall consider that the unknown surface lies among the set of surfaces ψ=const. Then φ and ψ characterize the mass flows of gas through curvilinear bounded sectors and layers, and therefore vary over limited ranges for stream tubes passing through the contours l_1 and l_2. In the space of the variables (p, φ, ψ) the region to be studied represents a rectangular region with boundaries parallel to the coordinate axes.

Let us consider a system of equations for the spatial flows of a nonviscous and non-heat-conducting gas with arbitrary thermodynamic properties [4]. The continuity equation $\operatorname{div}(\rho \mathbf{v}) = 0$, where ρ is the density and $\mathbf{v}$ is the velocity vector, is identically satisfied if the mass flow density vector $\rho\mathbf{v}$ is expressed in the form of the vector product of the gradient of the two scalar functions φ and ψ:

$$\rho \mathbf{v} = \nabla\varphi \times \nabla\psi.$$

A formal transformation to the independent variables (p, φ, ψ) in the Euler equations leads to the relationships

$$\frac{v_{1p}}{\dfrac{\partial(x_2, x_3)}{\partial(\varphi, \psi)}} = \frac{v_{2p}}{\dfrac{\partial(x_1, x_3)}{\partial(\varphi, \psi)}} = \frac{v_{3p}}{\dfrac{\partial(x_1, x_2)}{\partial(\varphi, \psi)}} = \lambda, \tag{2.1}$$

where v_i are the projections of the velocity on the coordinate axes x_i, the index p signifying partial differentiation with respect to p. From the definition of the functions φ and ψ we have the equations

$$x_{1p}/v_1 = x_{2p}/v_2 = x_{3p}/v_3; \; \lambda = 1. \tag{2.2}$$

*As in Russian original – Publisher.

Eliminating the projections of the velocities from (2.1) and (2.2), we obtain three equations in partial derivatives with respect to the functions $x_i(p, \varphi, \psi)$:

$$\frac{\partial(x_i, x_j)}{\partial(\varphi, \psi)} = (-1)^{i+j}\left(\frac{v x_{lp}}{\sqrt{\sum_{k=1}^{3}(x_{kp})^2}}\right)_p, \tag{2.3}$$

$$\frac{v^2}{2} + h = \text{const} = h_m,\ l = 6 - i - j,\ i < j.$$

Let us imagine a tube of rectangular cross section formed by four intersecting surfaces, two from each of the two families φ =const and ψ =const. In Fig. 2 the isobaric cross section of such a tube is shown shaded. If ΔS_l are the areas of the projections of the shaded cross section on the three coordinate planes and $\Delta q = \Delta\varphi \cdot \Delta\psi$ is the mass flow through the cross section of such a tube, then

$$\partial(x_i, x_j)/\partial(\varphi, \psi) \approx \Delta S_l / \Delta_q$$

and Eqs. (2.3) may be rewritten in the form

$$\Delta S_l / \Delta q \approx -v_{lp}. \tag{2.4}$$

When the axis of the tube is almost rectilinear and only the axial velocity component differs appreciably from zero, Eq. (2.4) is transformed into the ordinary relationship for one-dimensional flow in a channel with a variable cross-sectional area.

§3. We shall consider that the stream function $\psi = 0$ on the unknown stream surface. We shall seek all three unknown functions x_i in the form of polynomials with respect to ψ. In view of the abundance of whole-numbered parameters we shall denote the numbers of the coordinates and projections by the indices i, j, l. Such parameters will take only three discrete values: 1, 2, 3. We shall denote the numbers of the terms by k, m, n and shall write them, not as indices, but as arguments of the corresponding functions. The ranges of variation of these quantities will be specially indicated. The continuous arguments p and φ will not be written out. Under such conditions the form of the unknown approximating polynomials will be

$$x_i = \sum_n x_i(n)\psi^n.$$

Let us accept the rule of summation with respect to repeated arguments only, these being specially written out in this particular case. If indices, powers, or whole-numbered factors are repeated, no summation is executed. For example,

$$\left(\frac{\partial x_i}{\partial p}\right)^2 = \sum_n X_i(n)\psi^n, \text{ where } X_i(n) = \frac{\partial x_i(k)}{\partial p}\frac{\partial x_i(n-k)}{\partial p}\ (0 \leqslant k \leqslant n). \tag{3.1}$$

On the right-hand side of the latter equation summation takes place over the repeated argument k but not over the repeated index i. Analogously, for the square of the velocity projection

$$V_i = v_i^2 = \sum_n V_i(n)\psi^n;\ V_i(n) = v_i(k)v_i(n-k)\ (0 \leqslant k \leqslant n), \tag{3.2}$$

where $v_i(n)$ are the coefficients of the polynomial

$$v_i = \sum_n v_i(n)\psi^n.$$

It follows from (3.2) that

$$v_j(n) = [V_i(n) - v_i(k)v_i(n-k)]/2v_i(0)\ (1 \leqslant k \leqslant n-1); \tag{3.3}$$

on the other hand

$$v_i = \frac{v\dfrac{\partial x_i}{\partial p}}{\sqrt{\sum_{i=1}^{3}\left(\dfrac{dx_i}{\partial p}\right)^2}}. \tag{3.4}$$

For the square of the velocity modulus we write

$$v^2 \approx \sum_n w(n)\psi^n.$$

Squaring Eq. (3.4), we rewrite this in the form

$$\left[\sum_n V_i(n)\psi^n\right]\left\{\sum_n \sum_{i=1}^{3}[X_i(n)\psi^n]\right\} = \left[\sum_n w_i(n)\psi^n\right]\left[\sum_n X_i(n)\psi^n\right].$$

Cross-multiplying the sums and equating the coefficients of identical powers of ψ, by virtue of the foregoing summation rule we obtain

$$V_i(n-k)\left[\sum_{i=1}^{3} X_i(k)\right] = X_i(k)w_i(n-k)\ (0 \leqslant k \leqslant n).$$

From this follow the recurrence equations

$$V_i(n) = \frac{X_i(0)\,w(0) + X_i(k)\,w(n-k) - V_i(n-k)\left[\sum_{i=1}^{3} X_i(k)\right]}{\sum_{i=1}^{3} X_i(0)}\ (1 \leqslant k \leqslant n). \tag{3.5}$$

Let us proceed in a similar way in relation to the left-hand sides of the basic equations (3.3):

$$\frac{\partial x_i x_j}{\partial(\varphi, \psi)} = \sum_n a_{ij}(n)\psi^n,$$

where

$$a_{ij}(n) = (n-k+1)\left[\frac{\partial x_i(k)}{\partial\varphi}x_j(n-k+1) - \frac{\partial x_j(k)}{\partial\varphi}x_i(n-k+1)\right]\ (0 \leqslant k \leqslant n), \tag{3.6}$$

and by reason of the equations of motion (3.3)

$$a_{ij}(n) = (-1)^{i+j}\frac{\partial v_l(n)}{\partial p},\quad l = 6 - i - j. \tag{3.7}$$

The latter equation recurrently determines the coefficients of the unknown functions; it follows from (3.6) and (3.7) that

$$\frac{\partial x_i(0)}{\partial\varphi}x_j(m+1) - \frac{\partial x_j(0)}{\partial\varphi}x_i(m+1) = \frac{1}{m+1}\left\{(-1)^{i+j}\frac{\partial v_l(m)}{\partial p} - \right.$$

$$\left. -(m-k+1)\left[\frac{\partial x_i(k)}{\partial\varphi}x_j(m-k+1) - \frac{\partial x_j(k)}{\partial\varphi}x_i(m+1-k)\right]\right\} = (-1)^{i+j}\beta_{ij}(m)\ (1 \leqslant k \leqslant m); \tag{3.8}$$

$\beta_{ij}(m)$ represent the right-hand sides of the resultant equations not depending on the leading coefficients $x_i(m+1)$. Thus, these coefficients should satisfy the linear system of equations (3.8), which is characterized by the following matrix of coefficients:

$$\left\|\begin{matrix} \frac{\partial x_2(0)}{\partial\varphi} & -\frac{\partial x_1(0)}{\partial\varphi} & 0 & \beta_{12}(m) \\ \frac{\partial x_3(0)}{\partial\varphi} & 0 & \frac{\partial x_1(0)}{\partial\varphi} & -\beta_{13}(m) \\ 0 & \frac{\partial x_3(0)}{\partial\varphi} & -\frac{\partial x_2(0)}{\partial\varphi} & \beta_{23}(m) \end{matrix}\right\|.$$

The rank of the matrix should be equal to two, since this is the rank of the basic determinant. If we assume that all the derivatives with respect to φ differ from zero, then the last requirement is equivalent to the equation

$$[\partial x_1(0)/\partial\varphi]\beta_{23}(m) + [\partial x_2(0)/\partial\varphi]\beta_{13}(m) + [\partial x_3(0)/\partial\varphi]\beta_{12}(m) = 0.$$

This equation imposes a relationship upon the coefficients with order numbers smaller than $m+1$, in particular, for $m=0$, on the functions $x_i(0)$ and their derivatives with respect to p and φ. Hence only two of these functions may be regarded as arbitrary. If the solution is constructed in the form of a polynomial in ψ up to terms of the order of $m+1$, then we may take any of the functions $x_i(m+1)$ as the third independent arbitrary function, since among the equations (3.8) only two are linearly independent, so that the two leading coefficients are determined in terms of the third.

§4. As already indicated, a quality criterion for comparing the permissible stream surfaces is provided by certain characteristics of the mechanical (force) action of the flow on these surfaces. Let us apply the usual procedure for determining this action, based on the integrated application of the momentum law: Let S_1, S_2 be the surfaces of isobars based on the specified contours l_1 and l_2, and let S be the stream surface passing through the same contours. The closed surface comprising the three parts indicated is taken as the control surface. At the initial instant this surface delimits the volume of gas to which the momentum law is applied. Obvious considerations lead to the equation

$$-\iint_S p\cos(\mathbf{n}, x_i)\,\partial S - \iint_{S_1} p\cos(\mathbf{n}, x_i)\,dS - \iint_{S_2} p\cos(\mathbf{n}, x_i)\,dS = \\ = \iint_{S_2} \rho v_i v_n dS - \iint_{S_1} \rho v_i v_n dS \quad (i=1, 2, 3), \tag{4.1}$$

where $\mathbf{n}$ is the external normal to the corresponding surface; v_n is the projection of the velocity on this normal. Let R_i be the projection of the force acting between the flow and the surface S on the axis with number i:

$$R_i = \iint_S p\cos(\mathbf{n}, x_i)\,dS.$$

Remembering that $\cos(\mathbf{n}, x_l)dS = dx_i dx_j$ $(l = 6 - i - j,\ i \neq j)$, Eq. (4.1) may be written as follows:

$$R_i = \sum_{j=1}^{3} (I_{ij}|_{p=p_1} - I_{ij}|_{p=p}),$$

where

$$I_{ij} = \iint (p\delta_{ij} + \rho v_i v_j)\frac{\partial(x_k, x_m)}{\partial(\varphi, \psi)}\,d\varphi d\psi, \tag{4.2}$$

$k < m$, $j \neq k$, $i \neq k$, and all the indices may take the values 1, 2, 3; δ_{ij} is the Kronecker symbol.

The foregoing recurrence equations (3.1), (3.3), (3.5), and (3.8) determine the coefficients of the integrand polynomial in (4.2):

$$(p\delta_{ij} + \rho v_i v_j)\frac{\partial(x_k, x_m)}{\partial(\varphi, \psi)} \approx \sum_{n=0}^{N} K_{ij}(n)\,\psi^n.$$

Hence

$$I_{i,j} \approx \int_{\varphi_1}^{\varphi_2} \sum_n K_{ij}(n)\frac{\psi_0^{n+1}}{n+1}\,d\varphi.$$

Here ψ_0 is the characteristic of the mass flow through the stream tube bounded by the surface S:

$$R_i \approx \int_{\varphi_1}^{\varphi_2} \sum_{n=0}^{N} \frac{\psi_0^{n+1}}{n+1}\left[\sum_{j=1}^{3}(K_{ij}|_{p=p_1} - K_{ij}|_{p=p_2})\right]d\varphi; \tag{4.3}$$

i.e., the optimized action is sought in the form of a polynomial in powers of the mass of gas responsible for this action. By virtue of Eqs. (3.2), (3.8), (3.5), and (3.3) the integrand in Eq. (4.3) depends on the derivatives $\partial^n x_i(0)/\partial p^n$ of all orders up to $2(N+1)$ inclusively. These derivatives may be regarded as unknown functions of φ, since they are calculated for fixed values of $p = p_1$ and $p = p_2$; i.e., the total number of unknown functions is $4(N+1)$. Apart from these actual functions, their derivatives with respect to φ also enter into the integrand of (4.3). Thus after the solution of the extremal problem for the integral (4.3) the unknown stream surface is determined by approximate representations of the functions $x_i(0)$ in the form of polynomials in p of degree $2(N+1)$ based on the two ends with contours l_1 and l_2. In general, unless additional conditions of smoothness are applied, a break occurs in the overall surface where the surfaces arising from the two ends join. Generally speaking, we cannot demand continuity of the unknown functions with respect to φ either (for example, in the simplest case of the so-called V-shaped bodies the contours l_1 and l_2 are triangles).

For $N=0$ a linear relationship is obtained between the integrals expressing the flows of momentum through the isobaric surfaces (analogs of the function σ) and the corresponding mass flows. According to the foregoing discussion this case should be considered as a generalization of the principal approximate methods to spatial problems, the specific formulation of which involves the application of additional conditions according to the particular method chosen. The integrand of Eq. (4.3) contains $x_i(1)$; i.e., the case $N=0$ corresponds to representations of the coordinates linear in ψ. If for the same additional conditions we choose $N > 0$, then the following approximations, starting from $x_i(2)$, may be considered as corrections to the methods employed, and their relative values enable us to assess the validity of the procedures in question.

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SPECTRAL CHARACTERISTICS OF THE PULSATION EFFECT OF A PLANE TURBULENT JET ON A SOLID SURFACE

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INTRODUCTION

Turbulent processes in jets interacting with solid surfaces attract the attention of many researchers at this time [1-4]. Fundamental difficulties in both computational and experimental investigations hence arise in studying turbulence in the most important flow domain directly at the surface. Information about pressure fluctuations at the surface is an additional source of knowledge about the structure of turbulence in this domain. Moreover, data about turbulent near-wall pressures also have a direct applied value, related mainly to problems of computing the vibrations of structure elements [5].

§1. The present experiment was conducted with plane air jets issuing from a slot nozzle of width $d = 15$ mm (length 350 mm) at velocities of 75-220 m/sec. The Reynolds numbers hence varied in the range $0.7 \cdot 10^5$-$2.1 \cdot 10^5$, and the Mach number, in the range 0.2-0.64. The jet impinged on the flat surface of a massive turntable at distances of 360-640 mm from the nozzle exit. Modules with piezoelectric pressure fluctuation converters, similar to those described in [6], were mounted flush with the working surface of the slab. Spectral analysis of the signals from the converters was accomplished by an SI-1 spectrometer in three-octave frequency bands in the 50-10,000-Hz range. Transducers with a 1.3-mm-diameter detection surface had a practically constant response of about $4\mu V/Pa$ with respect to the frequency in this range. In order to check the vibration interference, the vibration response of the transducers was determined and the slab vibrations were measured during the tests.

§2. It was clarified in an analysis of the measurement results that the governing parameters of the fluctuating effect of the jet on the surface perpendicular to the jet at a distance x from the nozzle exit are the mean characteristics (the density ρ, the axial velocity v, and the width 2b) of the equivalent free jet at a distance x. This agrees with the information that the zone of interaction in the case under consideration extends a distance on the order of the nozzle width along the normal to the surface [7, 8]. Therefore, the flow in this domain should be determined by the free jet characteristics at a distance on the order of $x - d$ or, for $x \gg d$, at a distance x from the nozzle exit. It is seen from Fig. 1 that the values of the spectral density of the pressure fluctuations $\Phi/\rho^2 v^3 b$, reduced to dimensionless form, are functions of the reduced frequency $\omega b/v$ and the relative removal y/b from the jet axis in the whole range of velocities and distances x investigated. Three cases with different values of y/b are presented here: a) 0; b) 0.73; c) 2.7. The numbers correspond to the following values of v_0 in m/sec and x/d: 1) 75, 24; 2) 75, 33; 3) 75, 43; 4) 117, 24; 5) 117, 33; 6) 117, 43; 7) 218, 24; 8) 218, 33; 9) 218, 43.

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